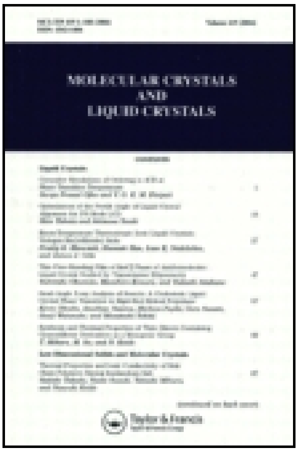

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Spin Glass State and Other Magnetic Structures with Their Symmetries in Terms of the Fibre Bundle Approach

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Spin Glass State and Other Magnetic Structures with Their Symmetries in Terms of the Fibre Bundle Approach

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The fibre bundle approach has been applied to derive the explicit formulas presenting all the eight fundamental magnetic structures and their symmetry groups including the spin glass state. The explanation of the uniqueness of the spin glass state has its roots in the appearance of the probability function p (i, j) *in the second term of the assumed Hamiltonian. This term actually describes the random distribution of either dopants or defects in the ferromagnetic matrix under the percolation threshold. On this basis the Gaussian type randomness was derived for both the general global magnetic coupling constant and the magnetization vector, the latter effect bringing to the statistical features of the magnetic structure and the magnetic symmetry group of the spin glass state.*

Keywords spin glass state; magnetic structures; magnetic symmetry groups; fibre bundle approach

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1. Introduction

To describe the symmetry of the magnetic structures as well as all other aperiodic structures one needs to formulate the corresponding symmetry groups whose action on these structures leaves them invariant. Several attempts in this respect have been undertaken, e. g. spin groups [1-3] or generalized color groups [4–6]. The extension of the spin groups to the description of quasicrystals is presented in [7]. The fibre bundles and their topological structures present a kind of a generalization of the Cartesian product of two spaces with arbitrary dimensions. Moreover this approach remembers to some extent the wreath groups concept introduced by Litvin [8–10], because a wreath group acts in a similar way as a structural group of the fibre bundle. As concerns the spin glass state, the authors have earlier proved that a certain minimum magnetic field appearing spontaneously is necessary for the stability of the spin glass state [11].

2. Fibre Bundle Approach

As the fibre bundle approach will be applied here to the description of the symmetry of magnetic structures let us recall a few definitions. A fibre bundle consists of [12]:

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- E – total space,
- B – base manifold,
- **E** – fibre (a kind of space, which is ascribed to every point of B and is "parametrized" by this point),
- G – a structural group which acts in a given fibre **E**,
- projection $\pi : E \to B$,
- section of $E : s : B \to E$ if $\pi \circ s = id$.

The fiber bundle is generalization of the Cartesian product of two vector spaces. The total space E is a manifold which locally looks like a product of the fibre **E** and of open sets, which cover the base manifold B.

3. Structure of the Bundle

In this section let us present the structure of a vector bundle serving to the description of the magnetic structures. The main idea of this structure consists in the use of the geometrical language of the vector bundles. It is well known that the vector bundles create the frames of the Standard Model of the elementary particles because they make a useful tool of the description of the fundamental elementary interactions. It seems that the application of the vector bundles to the description of the magnetic structures makes possible the unification of this description for all the structures and their symmetries.

The base manifold B of the six-dimensional vector bundle E_6 makes here the space filled with a magnetic system represented by 2-dimensional lattices of atoms creating magnetic planes P_n, where n numerates the consecutive planes. Thus the base manifold B is sum:

$$B = \underset{n}{\cup} P_n. \tag{1}$$

Each of these planes possesses a constant magnetization vector $\mathbf{M}_n$. With each such plane P_n is connected a fibre **E**, which is a 3-dimensional vector space. Hence the total space of the bundle E_6 assumes locally the form:

$$P_n \times \mathbf{E}. \tag{2}$$

The transition functions ϕ_{mn} define a mapping:

$$\phi_{mn} : (P_m \cap P_n) \times \mathbf{E} \to (P_m \cap P_n) \times \mathbf{E} \tag{3}$$

and can be determined by giving for a vector $\mathbf{r} \in P_m \cap P_n$ the linear mapping $t_{mn}(\mathbf{r})$ acting in the fibre **E** :

$$\phi_{mn}(\mathbf{r}, \mathbf{e}) = (\mathbf{r}, t_{mn}(\mathbf{r})\,\mathbf{e}), \tag{4}$$

where $\mathbf{e} \in \mathbf{E}$. The mappings $t_{mn}(\mathbf{r})$ meet the folowing conditions:

1. $t_{mm}(\mathbf{r}) = I_3$ (unit matrix 3×3),
2. $t_{mn}(\mathbf{r}) = t_{nm}^{-1}(\mathbf{r})$,
3. if $\mathbf{r} \in P_m \cap P_n \cap P_r$, then: $t_{mn}(\mathbf{r})\, t_{nr}(\mathbf{r}) = t_{mr}(\mathbf{r})$ (cocycle condition).

The section of the bundle E_6 is a mapping from the base space B to the total space E_6. In the case under consideration the section is defined by the magnetization vector field

$\mathbf{M}(\mathbf{r})$ for every $\mathbf{r} \in B$. It means that to every point belonging to B is assigned a vector $\mathbf{M}$ belonging to the three-dimensional vector space being the fibre $\mathbf{E}$.

The planes P_m covering the base manifold B do not cross for the seven following magnetic structures: f – ferromagnetic, a – antiferromagnetic, s – simple spiral, fs – ferromagnetic spiral, ss – skew spiral, t – transverse spin wave, l – longitudinal spin wave. The way of transition from one plane P_m to another plane P_n determines the symmetry group of the system. In this case the transition functions t_{mn} create the symmetry group, which is simultaneously the group G of the bundle structure. In other words G is a structural group which acts in the standard fibre $\mathbf{E}$.

In case of the spin glass state the planes P_m cross creating the common parts as a result of the random fluctuations of the system. Thus for the spin glass state a space of the global magnetic coupling constant has been introduced (see Section 7) as the representative quantity of this state. This space makes the fibre. The symmetry group of the structure of such a bundle is group $SO(2)$, whose group elements are functions of the points of the base space. Group $SO(2)$ creates here a local gauge group.

4. The Extension of the Magnetic Moments to the Continuity Mode

The magnetization vector $\mathbf{M}$ after a certain modification can be interpreted as a section of the vector bundle E_6. This section should have a special form because in the physical system the magnetic moments are localized on the magnetic atoms and they vanish outside these atoms in the ideal crystal. To make possible the use of the fibre bundles approach one has to assure that these sections (vectors $\mathbf{M}$) be continuous.

Let us first extend to the continuity mode a magnetic moment $\mathbf{m}_i$ of the i-th magnetic atom in the following way [13,14]:

$$\mathbf{m}(\mathbf{r}) = \mathbf{m}_i \exp\left[-(\mathbf{r} - \mathbf{r}_i)^2/\xi^2\right], \tag{5}$$

where $\mathbf{r}_i$ is the position vector of the i-th magnetic atom, ξ is a parameter of the characteristic scale of the atomic sizes.

Such a situation seems to be more physical, because in this case a magnetic moment presents a field which is significant only close to the crystal positions of the magnetic atoms and decreases outside very quickly.

Let us assume that the vector $\mathbf{M}_m(\mathbf{r})$ presents a vector sum per a unit volume of the atomic magnetic moments in a crystal sample of a given magnetic structure indexed by m. Thus

$$\mathbf{M}_m(\mathbf{r}) = \sum_{\mathbf{r}_n} \mathbf{M}_m(\mathbf{r}, \mathbf{r}_n), \tag{6}$$

where $\mathbf{r}_n$ is the coordinate of the n-th crystal plane and $\mathbf{M}_m(\mathbf{r}, \mathbf{r}_n)$ represents the magnetization of this plane.

Therefore according to the central theorem of the theory of probability (the Lyapunov theorem) the analogous formula to Eq. (5) is valid also for the vector $\mathbf{M}_m(\mathbf{r}, \mathbf{r}_n)$:

$$\mathbf{M}_m(\mathbf{r}, \mathbf{r}_n) = \mathbf{M}_m \exp\left[-(\mathbf{r} - \mathbf{r}_n)^2/d^2\right], \tag{7}$$

where d is a distance between the neighbour crystal planes.

The index m corresponds to the eight following structures: $m = f, a, s, fs, ss, t, l, sg$, where: f – ferromagnetic, a – antiferromagnetic, s – simple spiral, fs – ferromagnetic spiral, ss – skew spiral, t – transverse spin wave, l – longitudinal spin wave, sg – spin glass. The case of spin glass will be discussed below. The exponential factor guarantees here that the section $\mathbf{M}_m(\mathbf{r})$ is continuous and has a small value outside the crystal plane.

5. The Types of Eight Sections of the Fibre Bundle E_6

The types of these sections, which represent magnetic structures or in other words the magnetization vectors $\mathbf{M}_m(\mathbf{r})$ are given below [13]:

1. Ferromagnetic structure (f):

$$\mathbf{M}_f = M\widehat{\mathbf{e}}_1, \tag{8}$$

2. Antiferromagnetic structure (a):

$$\mathbf{M}_a = (-1)^n M\widehat{\mathbf{e}}_1, \tag{9}$$

3. Simple spiral (s):

$$\mathbf{M}_s = M\cos(n\phi)\widehat{\mathbf{e}}_1 + M\sin(n\phi)\widehat{\mathbf{e}}_2, \tag{10}$$

where ϕ is the spiral angle,
4. Ferromagnetic spiral (fs):

$$\mathbf{M}_{fs} = M\cos(n\phi)\widehat{\mathbf{e}}_1 + M\sin(n\phi)\widehat{\mathbf{e}}_2 + M\sin\psi\widehat{\mathbf{e}}_3, \tag{11}$$

where ψ is the angle between the vector $\mathbf{M}$ and the plane $(\widehat{\mathbf{e}}_1, \widehat{\mathbf{e}}_2)$,
5. Skew spiral (ss) :

$$\begin{aligned}\mathbf{M}_{ss} = {} & M\left[\cos^2(n\phi) + \sin^2(n\phi)\cos\theta\right]\cos(n\phi)\widehat{\mathbf{e}}_1 \\ & + M\left[\cos^2(n\phi) + \sin^2(n\phi)\cos\theta\right]\sin(n\phi)\widehat{\mathbf{e}}_2 \\ & + 2M\sin(n\phi)\sin\frac{\theta}{2}\sqrt{\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\cos^2(n\phi)}\widehat{\mathbf{e}}_3,\end{aligned} \tag{12}$$

where θ is the angle between the plane $(\widehat{\mathbf{e}}_1, \widehat{\mathbf{e}}_2)$ and the plane of the spiral; in other words θ is the skew angle and $\theta < \pi/2$.

As concerns the spin waves, let us consider them as the frozen waves in the crystal. Therefore no matter whether we deal with the commensurate or incommensurate spin waves one can represent these waves as follows:

6. Transverse spin wave (t):

$$\mathbf{M}_t = f(n)M\widehat{\mathbf{e}}_1, \tag{13}$$

where $f(n)$ is the periodic function with the period equal to the spin wavelength. Note that if the latter period is commensurate with the crystal periodicity one can say that the spin wave is commensurate, and vice versa the incommensurability of $f(n)$ points to the incommensurability of the spin wave. Therefore if n numerates the crystal planes, then $f(n)$ represents the values of this function on these planes. The above remarks apply also to the longitudinal spin waves:

7. Longitudinal spin wave (l):

$$\mathbf{M}_l = f(n)M\widehat{\mathbf{e}}_3. \tag{14}$$

Note that the general formula concerning all the three spirals mentioned above, i.e. items 3., 4. and 5., can be expressed as follows:

$$\begin{aligned}\mathbf{M}_A(\phi,\theta,\psi) &= M\left[\cos^2(n\phi)+\sin^2(n\phi)\cos\theta\right]\cos(n\phi)\widehat{\mathbf{e}}_1\\ &\quad + M\left[\cos^2(n\phi)+\sin^2(n\phi)\cos\theta\right]\sin(n\phi)\widehat{\mathbf{e}}_2\\ &\quad + M\left(2\sin(n\phi)\sin\frac{\theta}{2}\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\cos^2(n\phi)}+\sin\psi\right)\widehat{\mathbf{e}}_3,\end{aligned} \tag{15}$$

where $A = s, fs, ss$. Thus the three angles ϕ, θ, ψ describe in an unambiguous way all the three spirals, namely s corresponds to $0 < \phi < \pi$, $\theta = 0$, $\psi = 0$, fs corresponds to $0 < \phi < \pi$, $\theta = 0$, $\psi < \pi/2$, ss corresponds to $0 < \phi < \pi$, $\theta < \pi/2$, $\psi = 0$.

8. Spin glass (sg):

$$\mathbf{M}_{sg}(\mathbf{r}) = \frac{1}{\sqrt{(2\pi)^3}}\sum_i(\sigma_i)^{-2/3}\,\mathbf{m}_i\exp\left[-\frac{(\mathbf{r}-\mathbf{r}_i)^2}{2\sigma_i^2}\right], \tag{16}$$

(see below Section 7, Eq. (40)).

The randomness of $\mathbf{M}_{sg}(\mathbf{r})$ seems to be of a Gaussian type, because it is a sum of many other random quantities, the subject of the central limit theorem of the theory of probability (the Lyapunov theorem). Therefore the above formula corresponds to the following distribution function of $\mathbf{M}_{sg}$:

$$F(\mathbf{M}_{sg}) = \frac{1}{\sqrt{(2\pi)^3}}(\Sigma)^{-2/3}\exp\left[-\frac{(\mathbf{M}_{sg}-\mathbf{M}_0)^2}{2\Sigma_i^2}\right], \tag{17}$$

where $\mathbf{M}_0$ is the average value of magnetization and Σ is the variance of the Gauss distribution.

6. Total Magnetic Symmetry Groups Corresponding to the Above Sections (Magnetic Structures)

A total magnetic symmetry group can then be defined as a group which remains $\mathbf{M}_m(\mathbf{r})$ invariant. This group consists of two factors: G_m, which is defined below, and G_Λ, which presents the symmetry group of the crystalline structure [13].

A magnetic symmetry group G_m is defined here by the invariance of the above vectors $\mathbf{M}_m(\mathbf{r})$ with respect to some elements of the Euclidean group $e(3)$ of the three-dimensional space V_3.

Thus the magnetic group G_m is defined for the above structures by the condition:

$$G_m = \{g \in e(3) | g \cdot \mathbf{M}_m = \mathbf{M}_m\}, \tag{18}$$

where the dot $\cdot$ means the action of g on $\mathbf{M}_m$:

$$g \cdot \mathbf{M}_m = A\mathbf{M}_m + \mathbf{t}, \quad g = (A, \mathbf{t}). \tag{19}$$

A is a rotation matrix and $\mathbf{t}$ is a translation vector and is taken as modulo lattice vector. Let us find the explicit forms of the symmetry groups for the above eight magnetic structures [13].

1. The magnetic group G_f for the ferromagnetic structure. This group has to conserve $\mathbf{M}_f = M\widehat{\mathbf{e}}_1$:

$$g \cdot \mathbf{M}_f = A\mathbf{M}_f + \mathbf{t}. \tag{20}$$

Thus:

$$A\widehat{\mathbf{e}}_1 = \widehat{\mathbf{e}}_1, \qquad \mathbf{t} = 0. \tag{21}$$

From these conditions it follows that the matrix A has the form:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & c & d \end{pmatrix} \tag{22}$$

where $ad - bc = 1$. This magnetic group in the space V_3 has the form:

$$G_f = \left\{ \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & c & d \end{pmatrix}, 0 \right) \text{ and } ad - bc = 1 \right\}. \tag{23}$$

The total magnetic group $\widehat{G}_f$ in the space E_6 is the tensor product of G_f and the space group G_Λ of the crystal structure:

$$\widehat{G}_f = G_f \otimes G_\Lambda. \tag{24}$$

2. For the antiferromagnetic structure one can obtain:

$$G_a = \left\{ \left(\begin{pmatrix} (-1)^n & 0 & 0 \\ 0 & a & b \\ 0 & c & d \end{pmatrix}, 0 \right) \text{ and } ad - bc = 1 \right\}. \tag{25}$$

The index n numerates the consecutive crystal lattice planes; to each such plane a corresponding vector $\mathbf{M}$ is ascribed. The total magnetic group $\widehat{G}_a$ in the space E_6 is the tensor product of G_a and the space group G_Λ of the crystal structure:

$$\widehat{G}_a = G_a \otimes G_\Lambda. \tag{26}$$

3. For the simple spiral the symmetry group G_s in V_3 has to conserve:

$$g \cdot \mathbf{M}_s = A\mathbf{M}_s + \mathbf{t}. \tag{27}$$

This condition leads to the group G_s:

$$G_s = \left\{ \left(\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, 0 \right) \text{ and } \alpha = \phi \right\}. \tag{28}$$

Thus:

$$\widehat{G}_s = G_s \otimes G_\Lambda. \tag{29}$$

4. In the case of the ferromagnetic spiral the symmetry group G_{fs} in V_3 presents a product of the corresponding symmetry matrices for both the simple spiral and the ferromagnetic structure taking into account that this time the ferromagnetic component points the direction $\widehat{\mathbf{e}}_3$:

$$G_{fs} = \left\{ \left(\begin{pmatrix} a\cos\alpha + b\sin\alpha & -a\sin\alpha + b\cos\alpha & 0 \\ c\sin\alpha + d\sin\alpha & -c\sin\alpha + d\cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}, 0 \right) \text{ and } \alpha = \phi,\ ad - bc = 1 \right\}. \tag{30}$$

Note that the value of ψ is related to the values of a, b, c, d. Thus:

$$\widehat{G}_{fs} = G_{fs} \otimes G_\Lambda. \tag{31}$$

5. In the case of the skew spiral the symmetry group G_{ss} in V_3 presents a product of the corresponding symmetry matrices for both the simple spiral and the rotation of this simple spiral about the $\widehat{\mathbf{e}}_1$ axis of the angle θ:

$$G_{ss} = \left\{ \left(\begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha\cos\theta & \cos\alpha\cos\theta & -\sin\theta \\ \sin\alpha\sin\theta & \cos\alpha\sin\theta & \cos\theta \end{pmatrix}, 0 \right) \text{ and } \alpha = \phi \right\}. \tag{32}$$

$$\widehat{G}_{ss} = G_{ss} \otimes G_\Lambda.$$

6. and 7. Spin waves:

$$G_{t,l} = \{(0, \mathbf{h}) \text{ where } \mathbf{h} \text{ is the periodicity vector of the function } f \text{ in (13) or (14)}\}. \tag{33}$$

Thus:

$$\widehat{G}_{t,l} = G_{t,l} \otimes G_\Lambda. \tag{34}$$

8. Spin glass:

In the case of the spin glass the symmetry group G_{sg} in V_3 corresponds to the arbitrary precession of the vector $\mathbf{M}$ around the direction of the stabilizing magnetic field mentioned in Introduction for $\varphi = const$ (see Section 7). Hence $G_{sg} = SO(2)$. Note that this precession can be also quantized. Thus:

$$\widehat{G}_{sg} = G_{sg} \otimes G_\Lambda. \tag{35}$$

7. Spin Glass State: The Eighth Magnetic Structure and Its Symmetry in Terms of the Fibre Bundle Approach

To describe the spin glass state for the ferromagnetic matrix below the percolation threshold let us assume the following Hamiltonian with two terms [15]:

$$\widehat{H} = -\sum_{j\neq i} \sum_{i=1}^{N_j - n_j} J_{ij} \widehat{S}_i \cdot \widehat{S}_j - \sum_{j\neq i} \sum_{i=1}^{n_j} J'_{ij} p(i,j) \widehat{S}_i \cdot \widehat{S}_j, \tag{36}$$

where N_j is a number of magnetic ions surrounding a j-th magnetic ion and interacting with it with a coupling constant J_{ij}, n_j is a number of dopants (or defects) surrounding a j-th ion, J'_{ij} is coupling constant between i-th and j-th ion in the presence of a dopant (or a defect), $p(i, j)$ is probability of appearance of a dopant (or a defect) between i-th and j-th ions. The latter function corresponds to the situation below the percolation threshold [15,16]. Quantum-mechanical equation for magnetization $\mathbf{M}$ in the external magnetic field $H(\mathbf{r}, t) = h_0 \cos(\mathbf{q} \cdot \mathbf{r}) \cos(\omega t)$ oriented in direction z represents the time derivative $d\mathbf{M}(\mathbf{k}, t)/dt$ as a function of the parameter $\widetilde{J}(-\mathbf{q}')$ (see below) [17].

Let us denote $a, b, c = 1, 2, 3$. Thus this equation for the a-th component of the vector $\mathbf{M}$ looks like:

$$\frac{dM_a(\mathbf{k}, t)}{dt} = u\varepsilon_{abc} \sum_{\mathbf{q}'} \widetilde{J}(-\mathbf{q}')[\widehat{S}_c(\mathbf{k} + \mathbf{q}')\widehat{S}_b(\mathbf{q}')\rangle + \langle\widehat{S}_b(\mathbf{q}')\widehat{S}_c(\mathbf{k} - \mathbf{q}')\rangle] + \frac{1}{2} h_0 \varepsilon_{ab3}[M_b(\mathbf{k} + \mathbf{q}') + M_b(\mathbf{k} - \mathbf{q}')] \cos(\omega t), \tag{37}$$

where $u = g\mu/hV$ and the parameter $\widetilde{J}\left(-\mathbf{q}'\right)$:

$$\widetilde{J}\left(-\mathbf{q}'\right) = \sum_{j \neq i} \sum_{i=1}^{N_j - n_j} J_{ij} e^{i\mathbf{q}' \cdot (\mathbf{R}_i - \mathbf{R}_j)} + \sum_{j \neq i} \sum_{i=1}^{n_j} J'_{ij} p\,(i, j)\, e^{i\mathbf{q}' \cdot (\mathbf{R}_i - \mathbf{R}_j)}. \tag{38}$$

Assume that the external magnetic field $H(\mathbf{r}, t)$ is equal to zero. Therefore the second term on the right hand side of Eq.(37) vanishes.

$\widetilde{J}(-\mathbf{q}')$ itself turns out to be a Fourier transform of the magnetic coupling constants J_{ij} and J'_{ij} . Therefore $\widetilde{J}(-\mathbf{q}')$ can be interpreted as a kind of a "global" magnetic coupling constant. It is obviously a random quantity because of the probability function $p(i, j)$ appearing in the second term of the right hand side of Eq. (38).

The randomness of $\widetilde{J}(-\mathbf{q}')$ seems to be of a Gaussian type, because $\widetilde{J}(-\mathbf{q}')$ is a sum of many other random quantities, the subject of the central limit theorem of the theory of probability (the Lyapunov theorem). Therefore the distribution function $F(\widetilde{J})$ has the form:

$$F(\widetilde{J}) = \frac{1}{\sqrt{(2\pi)^3}} (\Sigma_J)^{-2/3} \exp\left[-\frac{\left(\widetilde{J} - \widetilde{J}_0\right)^2}{2\Sigma_J^2} \right], \tag{39}$$

where Σ_J is the variance of the Gauss distribution and $\widetilde{J}_0$ is the average value of $\widetilde{J}$.

By the way the random distribution $p(i, j)$ leads to the random distribution $p(J'_{ij})$ [18]. It causes the frustration of the magnetic couplings and the appearance of the spin glass state in the ferromagnetic matrix containing either dopants or defects.

The global magnetic coupling constant can be transformed by the Fourier transformation from the momentum space into position space conserving its Gaussian-like randomness. Taking into account its randomness one can say that the magnetization vector in the spin glass state becomes also a random quantity, which—after linearization of the quantum-mechanical equation for magnetization $\mathbf{M}$ – assumes also the Gaussian-like distribution (the Lyapunov theorem applies again). It is because during the operation of linearization the Gaussian mode of randomness is conserved. The latter concerns both the value and the direction of the magnetization vector.

Let us call φ the angle between the magnetization vector and the direction of the spontaneous homogeneous magnetic field which stabilizes the structure of the spin glass state [11]. Thus for $\varphi = const$ the energy of the system does not change.

In an isolated physical system with the structure of the spin glass state another effect could in principle keep also constant the energy of the system, for instance if any infinitesimal random fluctuation mentioned above of the value of the magnetization vector takes place, then—according to the law of conservation of energy—it has to be accompanied by the corresponding change of the angle φ compensating the change of energy caused by this fluctuation. The same line of reasoning is valid also vice versa, i.e. if someone assumes first the infinitesimal change of the angle φ.

In principle in every isolated physical system there could exist more such mechanisms that keep energy constant. One can then say that the magnetization vector is situated along a generatrix of a given cone whose axis coincides with the direction of a stabilizing magnetic field [11]. One can also say that any rotation (precession) of the magnetization vector at $\varphi = const$ around the direction of this stabilizing magnetic field makes the operation of symmetry of the spin glass state. As it was mentioned at the end of Section 5 the symmetry group of this precession is $SO(2)$.

Moreover such a precession could be modulated by the two effects mentioned above: the random Gaussian-like change of the value of the magnetization vector and the corresponding change of the angle φ (or the two effects in the reverse sequence). Both these effects compensate with respect to each other their influence onto the total energy of the system. Thus the total energy remains unchanged. As mentioned above the eventual quantization of this (modulated) precession can also be realized.

Fig. 1 presents the structure of the spin glass state with the magnetization vector oriented along a generatrix of a given cone whose axis coincides with the direction of this stabilizing magnetic field. According to the above remarks the magnetization vector $\mathbf{M}_{sg}(\mathbf{r})$ of the spin glass state in the point given by the position vector $\mathbf{r}$ is presented as follows:

$$\mathbf{M}_{sg}(\mathbf{r}) = \frac{1}{\sqrt{(2\pi)^3}} \sum_i (\sigma_i)^{-2/3} \mathbf{m}_i \exp\left[-\frac{(\mathbf{r}-\mathbf{r}_i)^2}{2\sigma_i^2}\right], \tag{40}$$

where $\mathbf{r}_i$ is the vector position of the i-th magnetic atom carrying the magnetic moment $\mathbf{m}_i$, σ_i is variance of the Gauss distribution.

8. Discussion

In the present paper a description of the magnetic structures and their symmetry groups in terms of fibre bundles has been introduced including the case of the spin glass state.

Such an approach turns out to be the most general, because it is based on the most general product of two arbitrary spaces, namely on the Cartesian product, which is very suitable to the combination of two "worlds", e.g. the "world of positions" ($\mathbf{R}^3$) and the "world of spins" (V_3) [14]. Thus the description of crystal structures is to be carried out in $\mathbf{R}^3$, whereas the description of spin structures is to be carried out in V_3.

The total magnetic group $\widehat{G}_m$ in the space E_6 is the tensor product of the magnetic group G_m and the space group G_Λ of the crystal structure, where $m = f, a, s, fs, ss, t, l, sg$, where: f – ferromagnetic, a – antiferromagnetic, s – simple spiral, fs – ferromagnetic spiral, ss – skew spiral, t – transverse spin wave, l – longitudinal spin wave, sg – spin glass.

A global magnetic coupling constant $\widetilde{J}(-\mathbf{q}')$ can be used for all the eight magnetic structures. As concerns the spin glass state, $\widetilde{J}(-\mathbf{q}')$ exhibits a randomness of the Gaussian

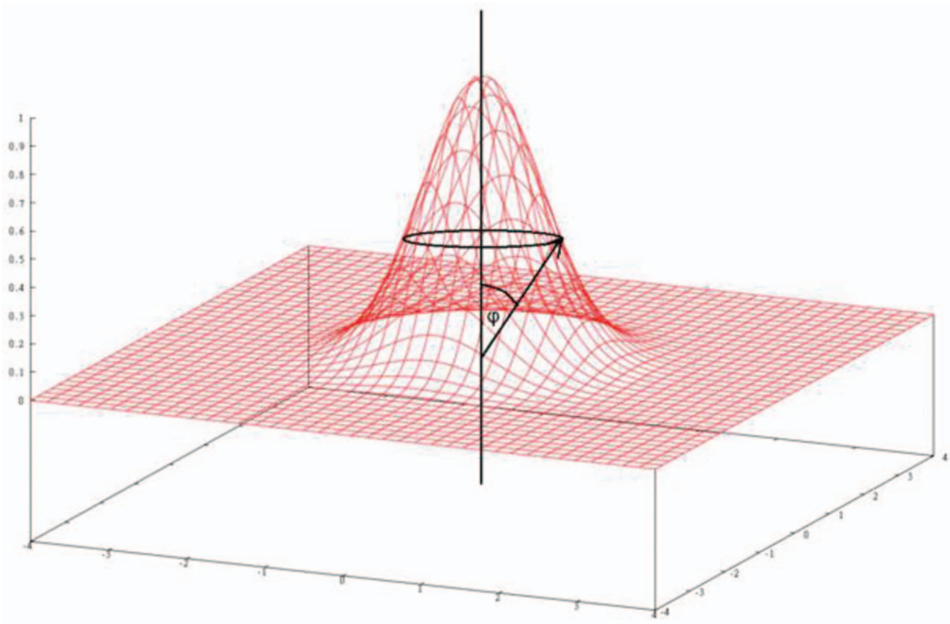

Figure 1. Structure of the spin glass state with the magnetization vector situated along a generatrix of a given cone whose axis coincides with the direction of the stabilizing magnetic field. This cone is parametrized by the angle φ . Any rotation (precession) of the magnetization vector at $\varphi = const$ around the direction of this stabilizing magnetic field makes the operation of symmetry of the spin glass state.

type because in this case the second term both in Eq. (38) and Eq. (36) does not vanish, whereas for all the remained seven magnetic structures it does.

It is the appearance of the probability function $p(i, j)$ in both these second terms which describes the random distribution of either dopants or defects in the ferromagnetic matrix under the percolation threshold. On this basis the Gaussian type randomness has been derived for both the global magnetic coupling constant and the magnetization vector, the latter effect bringing to the statistical features of the structure and the symmetry group of the spin glass state (see Sections 5 and 6).

Note that Eqs. (5–7) from one side and Eq. (40) from the other side have the same form although they have been obtained on the basis of quite different fundamental ideas. Thus for the first case it was rather intuition which led the authors to the Eqs. (5–7) via the extension of the individual magnetic moments to the continuity mode. In the second case Eq. (40) has been derived starting from the assumed special form of Hamiltonian (Eq. (36)) and with the use of the solution of the quantum-mechanical equation for magnetization in the external magnetic field (Eq. (37)) assumed to be equal to zero. In both cases use has been made of the central limit theorem of the theory of probability (the Lyapunov theorem).

Note that the second case—being scientifically more fundamental—gives approval to the first one. One has to add that σ_i in Eq. (40) corresponds to ξ in Eq. (5). Special emphasis has been put on the global magnetic coupling constant (Eq. (38)). Its interpretation has also been made.

Thus the fibre bundle approach equates the method of the symmetry analysis of magnetic structures with the method of the higher dimensional embeddings of the modulated structures. The symmetry groups appearing in the method of the symmetry analysis become structural groups of the bundles.

From the other side a higher dimensional space needed to the description of a modulated structure makes here the total space of the bundle. Thus these three methods, namely the symmetry analysis, the higher dimensional embeddings and the fibre bundles are equivalent.

The analogous situation is with the description of the magnetic structures with the use of the spin groups, where an additional type of symmetry operation is introduced [1–3]. This operation is represented by the time reversal operator r, which changes the sign of either the magnetic moment or spin. The time reversal operator there is represented by the $\mathbf{Z}_2$ group with two elements only: $-1, +1$.

This operator then acts as follows:

$$r \otimes G_\Lambda = G'_\Lambda = \{(r, \lambda)\}, \tag{41}$$

where λ belongs to G_Λ . In this description a general point in the four-dimensional space, where G'_Λ acts, is represented by x, y, z, s, and s is equal to either $+1$ or -1. In our approach the time reversal operator is already included into the groups G_m , where $m = f, a, s, fs, ss, t, l, sg$ as one of their elements.

Note that the Gaussian factor introduced above plays a double role: it makes the vector **M** to be a field and simultaneously makes the description of the magnetic structures more physical.

It seems that the fibre bundle approach could serve also for the description of the symmetry groups of all the other aperiodic structures, like e.g. the modulated nonmagnetic structures, quasicrystals (nonmagnetic and magnetic) etc.

It is worthwhile to mention here that these different magnetic structures have been found by the authors to be related with the values of certain topological invariants [19]. This will be the subject of other paper.

Literature

[1] Litvin, D. B. (1973). *Acta Cryst.*, *A29*, 651.
[2] Litvin, D. B., & Opechowski, W. (1974). *Physica*, *76*, 538.
[3] Litvin, D. B. (1977). *Acta Cryst.*, *A33*, 279.
[4] Koptsik, V. A., & Kotzev, I. N. (1974). Comm. Joint Institute Nuclear Research (Dubna, USSR)P4-8067, 8068,8466; P4-9964, 9965, (1976).
[5] Koptsik, V. A. (1975). *Krist. Tech.*, *10*, 231.
[6] Koptsik, V. A. (1978). *Ferroelectrics*, *21*, 499.
[7] Lifshitz, R. (1998). *Phys. Rev. Lett.*, *80*, 2717.
[8] Litvin, D. B. (1980). *Physica*, *101A*, 339.
[9] Litvin, D. B. (1980). *Annals of Israel Physical Society*, *3*, 371.
[10] Litvin, D. B. (1980). *Phys. Rev.*, *B21* 3184.
[11] Krok-Kowalski, J., Warczewski, J., Gusin, P., Śliwińska, T., Groń, T., Urban, G., Rduch, P., Władarz, G., Duda, H., Malicka, E., Pacyna, A., & Koroleva, L. I., (2009). *J. Phys.: Condens. Matter*, *21*, pp. 035402–035407.
[12] Sulanke, R., Wintgen, P. (1972). *Differentialgeometrie und Faserbündel*, Berlin.
[13] Warczewski, J., Gusin, P., Śliwińska, T., Urban, G., & Krok-Kowalski, J. (2007). *Central European Journal of Physics*, *5*(3), 377–384.
[14] Gusin, P., & Warczewski, J. (2010). *Mol. Cryst. Liq. Cryst. 521*, 288–292.

[15] Warczewski, J., Krok-Kowalski, J., Gusin, P., Duda, H., Fijak, J., Kozerska, K., Nikiforov, K., & Pacyna, A. (2003). *J. of Non-Linear Optics, Quantum Optics*, *30*, 301–320.
[16] Warczewski, J., Krok-Kowalski, J., Gusin, P., & Zajdel, P. (2005). *Journal of Physics and Chemistry of Solids*, *66*, 2044–2048.
[17] White, R. M. (1970). *Quantum Theory of Magnetism*, Mc Graw-Hill Book Company.
[18] Sherrington, D., & Kirkpatrick, S. (1975). *Phys. Rev. Lett.*, *35*, 1792–1795.
[19] Gusin, P., & Warczewski, J. (2004). *J. of Magn. and Magn. Mat.*, *28*(1/2-3), 178–187.